Engineering Notes

Minimum Swept-Wing Induced Drag with Constraints on Lift and Pitching Moment

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Nomenclature

A = coefficient for span-loading series
C_D = drag coefficient
C_L = lift coefficient
C_M = pitching-moment coefficient
ΔC_M = C_M contributed by the nonelliptical portion of the span loading
N = number of terms in the span-loading series
S = reference area for coefficients
a = aspect ratio
b = span
c = chord
c_l = section lift coefficient
c_m = section pitching-moment coefficient
m = mean aerodynamic chord
x, y = longitudinal and lateral coordinates
Λ = quarter-chord sweep angle
θ = transformed lateral coordinate, $\cos\theta = -2y/b$
λ = taper ratio

Subscripts

ac = airfoil section aerodynamic center
r = reference coordinate
0 = without lift

Introduction

AIRFOIL sections designed for high subsonic speeds frequently produce significant nose-down pitching moment. As a result, the lift-to-drag ratio of conventional airplanes employing such sections is reduced by the downward tail load required for equilibrium in pitch. However, the wing nose-down pitching moment of a swept-wing configuration can be reduced by increasing the loading on the inboard, forward portion of the wing and reducing the loading on the outboard, aft portion. This note determines the span loading of a swept wing that produces minimum induced drag with constraints on lift and pitching moment. The span-loading solution is then used to evaluate the minimum induced-drag penalty associated with the pitching-moment constraint.

Analysis

Glauert's truncated Fourier series for span loading[1] can be written

$$\frac{cc_l}{4b} = \sum_{n=1}^{N} A_{2n-1} \sin(2n-1)\theta \tag{1}$$

for symmetrical loading. In coefficient form, lift and induced drag are then given by

$$C_L = \pi a A_1 \tag{2}$$

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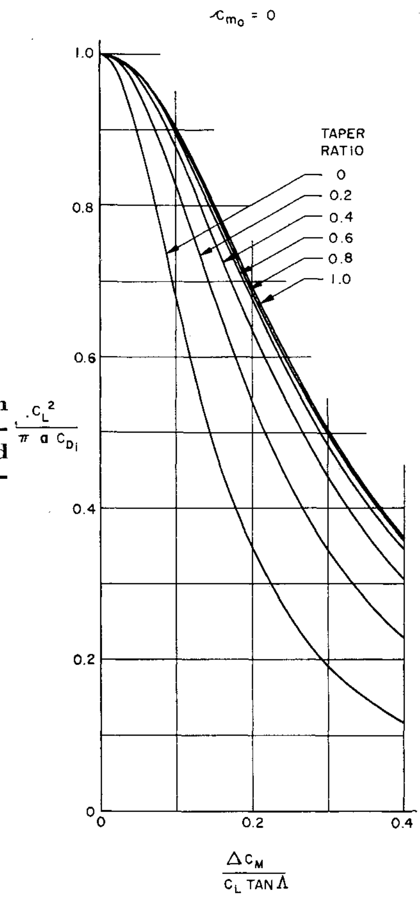

Fig. 1 Minimum induced-drag efficiency with C_L and C_M constaints (aspect ratio 4).

and

$$C_{D_i} = \pi a \sum_{n=1}^{N} (2n-1) A_{2n-1}^2 \tag{3}$$

Wing pitching-moment coefficient may be written as

$$C_M = \frac{1}{Sm}\int_{-b/2}^{b/2} c^2(y)\left[c_{m_0}(y) + c_l(y)\left(\frac{x_r - x_{ac}}{m}\right)\right] dy \tag{4}$$

The following assumptions are made: The section aerodynamic centers lie on the quarter-chord line, the wing-planform leading and trailing edges are straight, and Ref. 2 gives the length and lateral location of the mean aerodynamic chord For constant c_{m_0}, the pitching-moment coefficient becomes

$$C_M = c_{m_0} + 9\tan\Lambda\left[\frac{a(1+\lambda)}{1+\lambda+\lambda^2}\right]^2\left[\frac{\pi}{48}(1+2\lambda)A_1 + \frac{\pi}{64}(1-\lambda^2)(A_1+A_3) + \frac{1}{3}\left(1+\lambda-\frac{1}{2}\lambda^2\right)\times \sum_{n=1}^{N}\frac{(-1)^{n+1}A_{2n-1}}{(2n+1)(2n-3)}\right] \tag{5}$$

The minimum induced-drag problem may be developed from Eqs. (2, 3, and 5). The necessary condition for extremals[3] gives N linear equations in the N unknown Glauert coefficients and two unknown Lagrange multipliers. Eqs. (2) and (5) are also linear in the unknowns, completing a set of

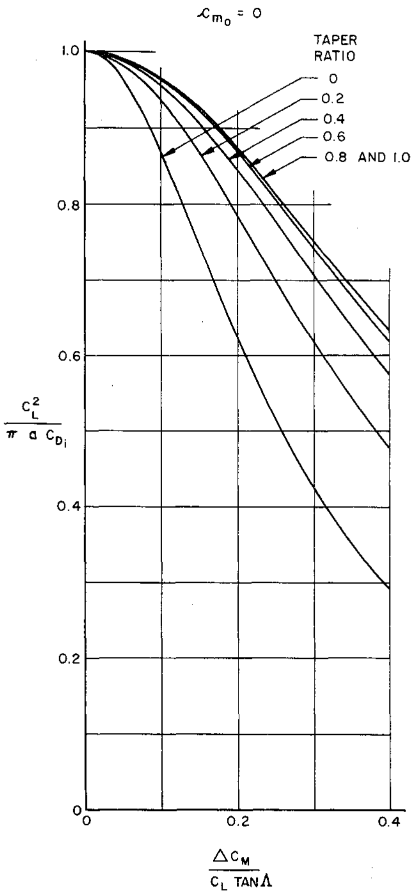

Fig. 2 Minimum induced-drag efficiency with C_L and C_M constraints (aspect ratio 7).

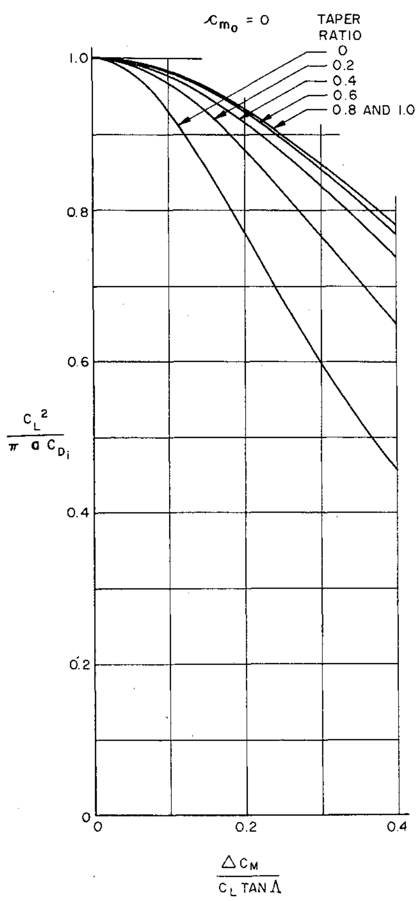

Fig. 3 Minimum induced-drag efficiency with C_L and C_M constraints (aspect ratio 10).

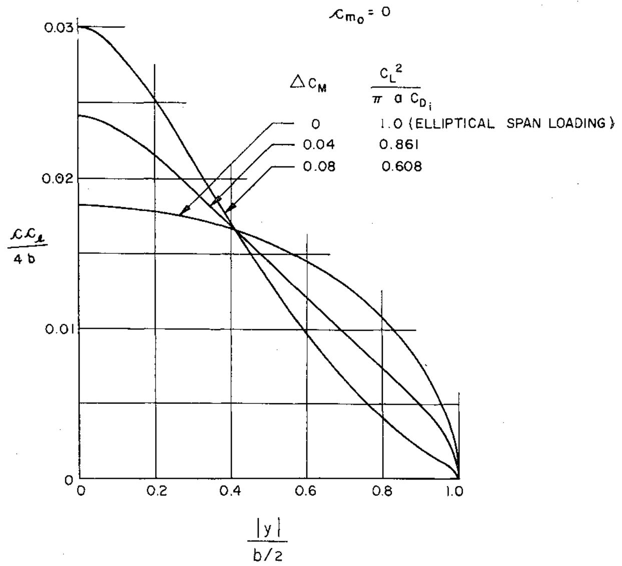

Fig. 4 Span loading for minimum induced drag. C_L and C_M constraints: $C_L = 0.4$, $N = 20$, $\Lambda = 30°$, $\lambda = 0.3$ (aspect ratio 7).

$N + 2$ linear, simultaneous equations that specify a unique extremal. The extremal satisfies the sufficiency condition of Ref. 3 for a minimum. The induced drag of the minimum is evaluated by Eq. (3).

Numerical Results

Figures 1–3 give the induced-drag efficiency parameter as a function of an incremental pitching-moment parameter for aspect ratios 4, 7, and 10. The parameter ΔC_M is the pitching-moment coefficient given by Eq. (5) when c_{m_0} and A_1 are zero. For configuration and force parameters typical of current subsonic jet transports at cruise ($a = 7$, $\Lambda = 30°$, $\lambda = 0.3$, $C_L = 0.4$), Fig. 2 indicates a minimum loss of induced-drag efficiency of 14% to produce $\Delta C_M = 0.04$. For the same configuration, Fig. 4 gives the span loading to produce minimum induced drag, subject to the C_L and C_M constraints, for varying ΔC_M. The large perturbations from elliptical span loading are the explanation for the significant losses of induced-drag efficiency.

Conclusions

The swept-wing span loading has been determined that produces minimum induced drag with constraints on lift and pitching moment. The numerical results indicate that small nose-down section pitching moments can be balanced by an appropriate span loading of a swept wing with small penalties in induced-drag efficiency. Large induced-drag penalties occur if the pitching-moment coefficient to be trimmed is greater than 0.02 for a configuration typical of swept-wing subsonic jet transports.

Viscosity and compressibility effects on wing drag have not been considered. Wing-tip and kink effects on chordwise location of airfoil section aerodynamic center have been ignored. In the writer's opinion, consideration of these effects is likely to reduce the moment that can be balanced by span-loading shifts for a given drag penalty.

References

[1] Glauert, H., *The Elements of Aerofoil and Airscrew Theory* (Cambridge University Press, London, 1948), 2nd ed., Chap. XI.

[2] Perkins, C. D. and Hage, R. E., *Airplane Performance, Stability and Control* (John Wiley & Sons, New York, 1949), pp. 89–92.

[3] Edelbaum, T. N., "Theory of maxima and minima," *Optimization Techniques with Applications to Aerospace Systems*, edited by G. Leitmann (Academic Press Inc., New York, 1962).